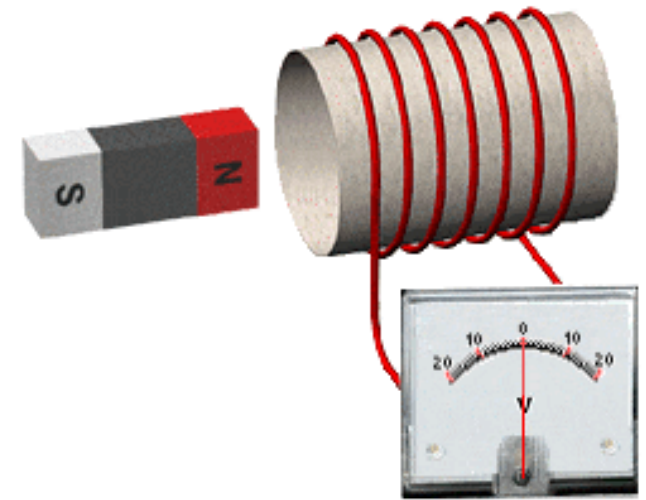


Faradays Law of Induction



Kieran Mckenzie

Physics – Grade 12 LS & GS

Unit Two – Electricity

Chapter 8 – Electromagnetic Induction



Think then Solve

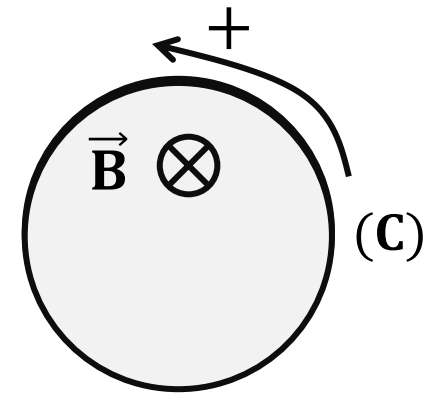
Exercise 1:

The aim of this exercise is to study the electromagnetic magnetic induction phenomenon.

For this aim, we consider a flat coil (C) of area $S = 5 \times 10^{-3} \text{ m}^2$, resistance $R = 10 \Omega$ and of 50 turns.

Part I: Variation of the magnetic field:

(C) is placed in a magnetic field \vec{B} & perpendicular to its plane as shown in document 1.



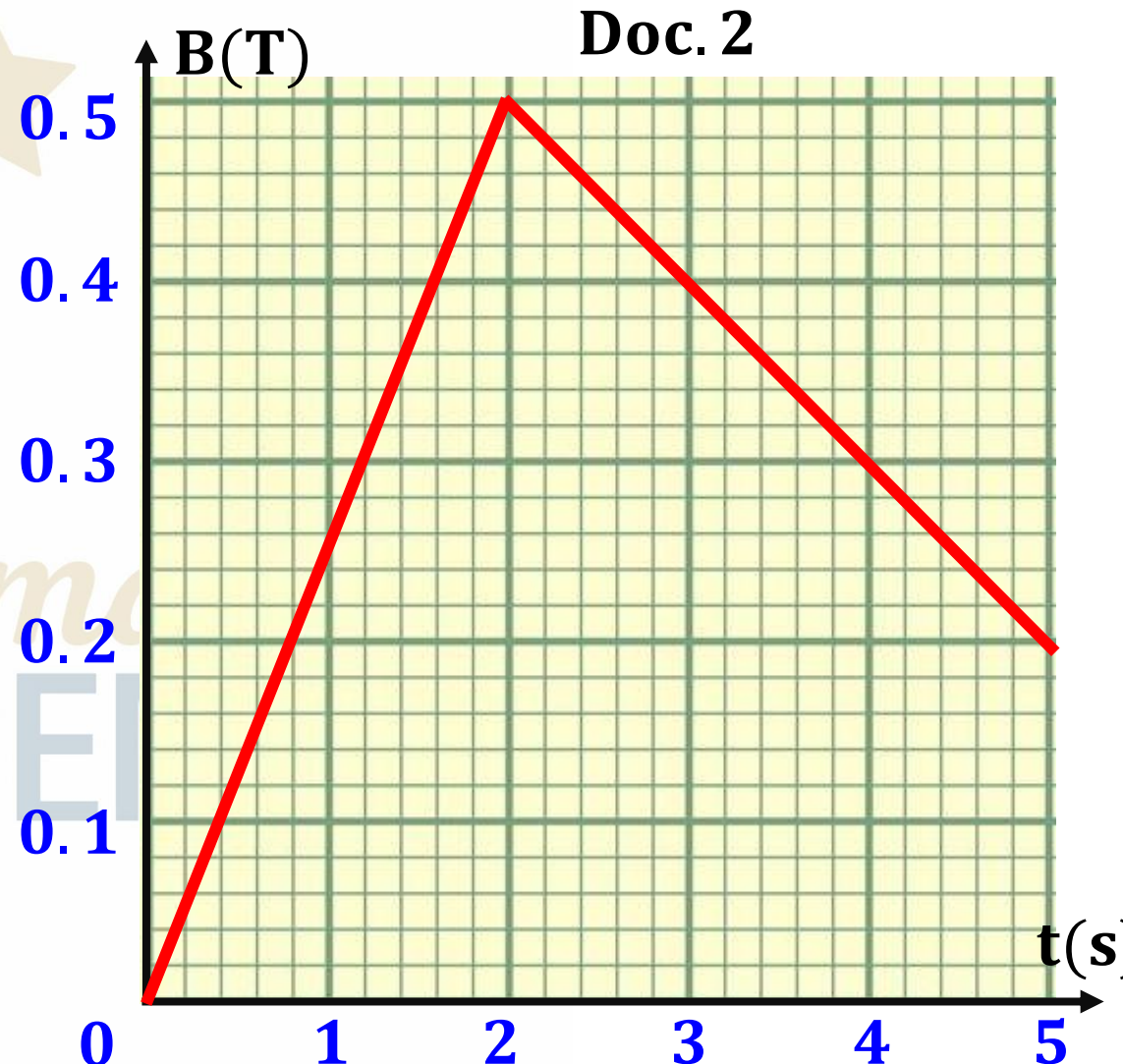
Doc. 1

Exercise 1:

Document 2 represents the variation of the intensity B of \vec{B} as a function of time.

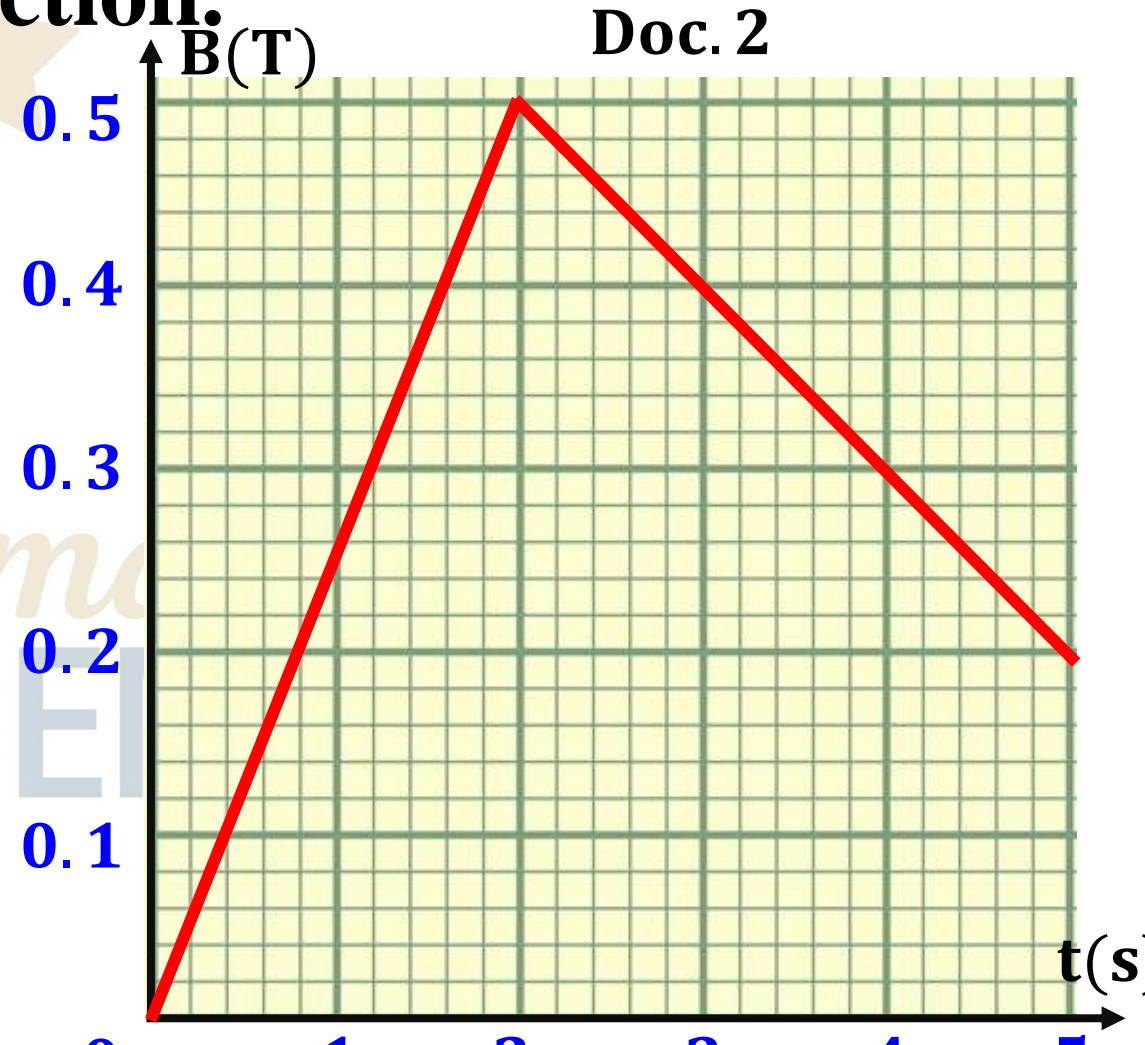
1. Explain the existence of an induced current i traversing (C).

2. Show that the expression of this induced current is in the form of $i = \frac{NS}{R} \times \frac{dB}{dt}$.



Exercise 1:

3. Calculate the values of i for the following $0 \leq t \leq 2s$ and for $2s \leq t \leq 5s$. Deduce its direction.
4. Apply Lenz's law to verify the results obtained in part 3.



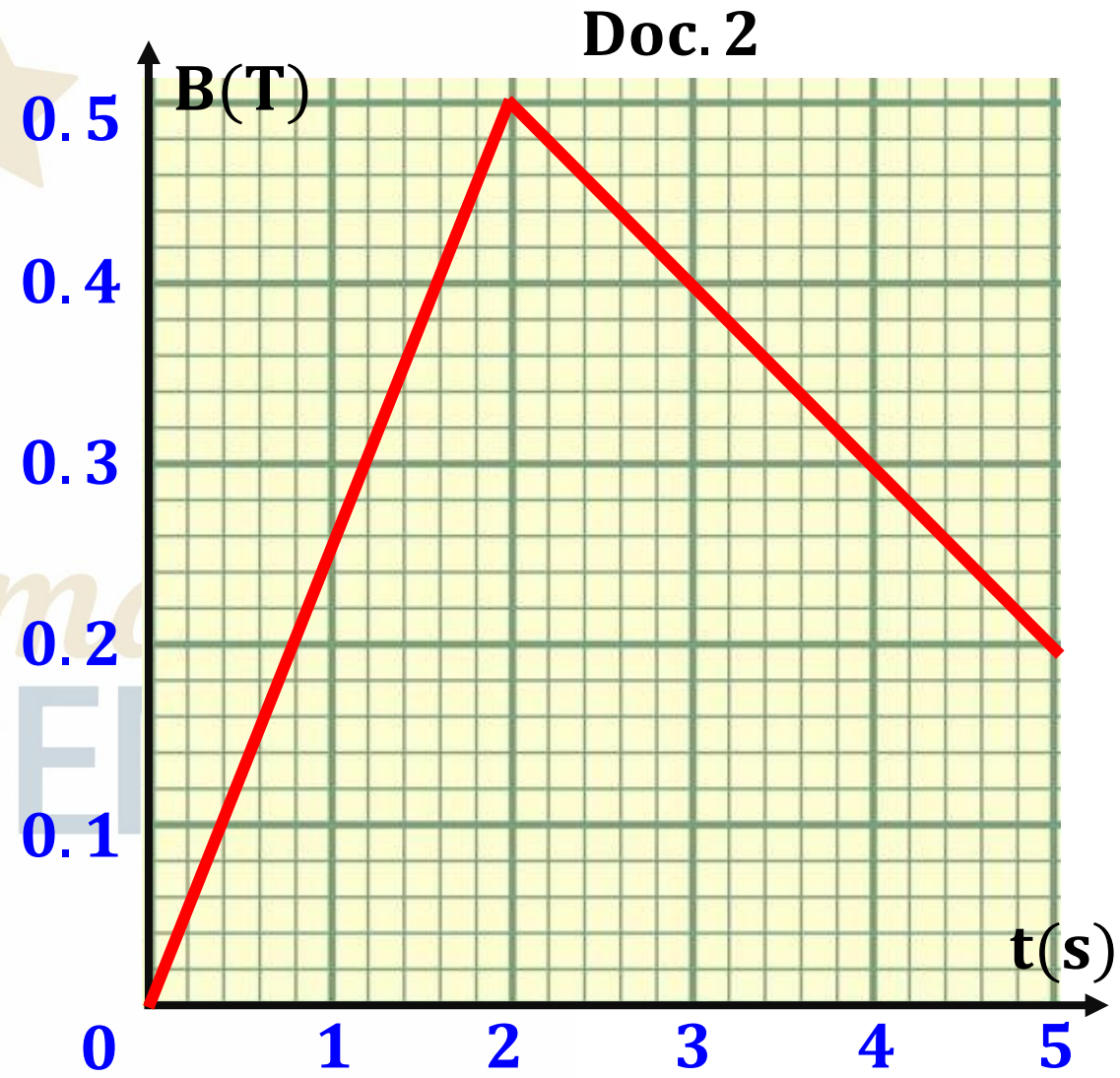
Exercise 1:

$S = 5 \times 10^{-3} \text{ m}^2$, $R = 10 \Omega$ and of 50 turns.

1. Explain the existence of an induced current i traversing (C).

The magnetic flux is variable because the magnetic field (\vec{B}) is variable.

The e.m.f “e” exist then the induced current flow in the flat coil (C).



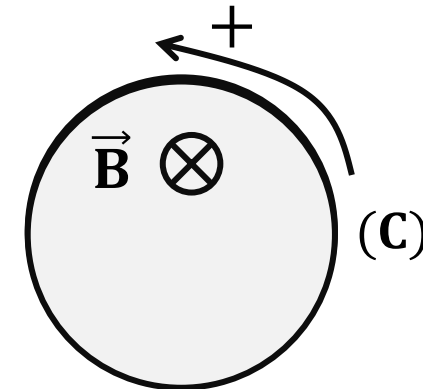
Exercise 1:

$S = 5 \times 10^{-3} \text{ m}^2$, $R = 10 \Omega$ and of 50 turns.

2. Show that the expression of this induced current

is in the form of $i = \frac{NS}{R} \times \frac{dB}{dt}$.

$$i = \frac{e}{R}$$



Doc. 1

$$\phi = NBS \cos(\vec{n}, \vec{B})$$

$$\phi = NBS \cos(180)$$

$$\phi = -NBS$$

$$i = \frac{NS \frac{dB}{dt}}{R}$$

$$e = -\frac{d\phi}{dt} \rightarrow e = +NS \frac{dB}{dt}$$

$$i = \frac{NS}{R} \times \frac{dB}{dt}$$

Exercise 1:

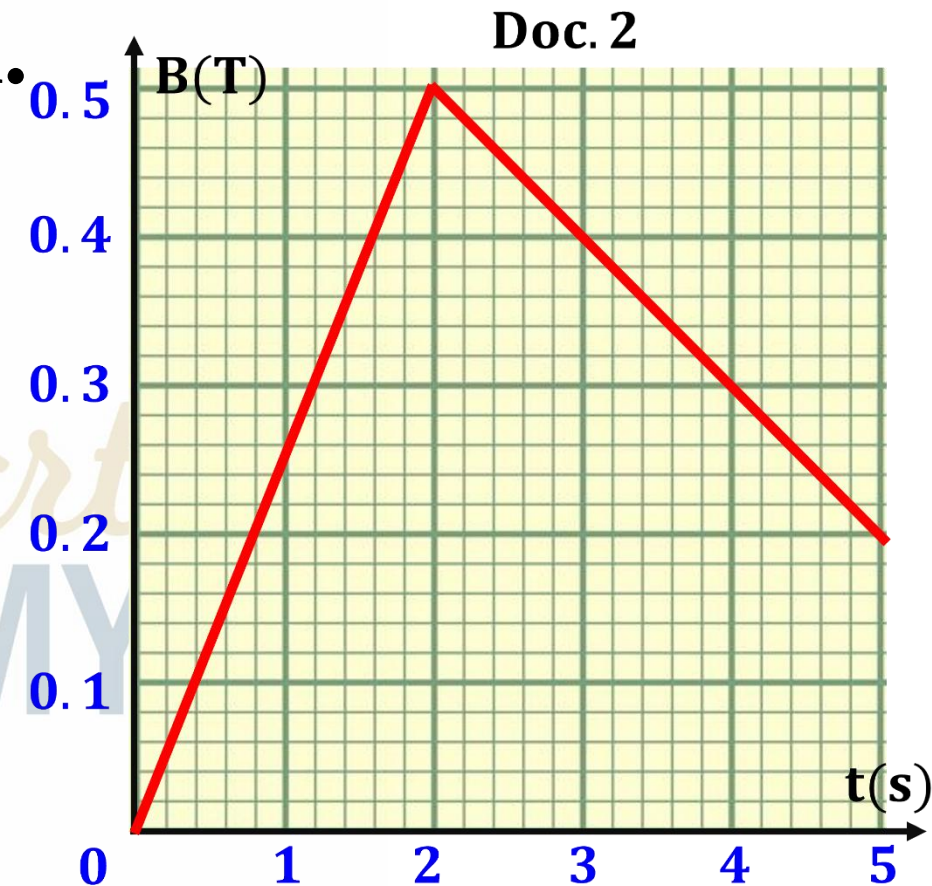
$S = 5 \times 10^{-3} \text{ m}^2$, $r = 10 \Omega$ and of 50 turns.

3. Calculate the values of i for the following $0 \leq t \leq 2\text{s}$ and for $2\text{s} \leq t \leq 5\text{s}$. Deduce its direction.

For $0 \leq t \leq 2\text{s}$

$$\text{slope} = \frac{dB}{dt} = \frac{B_2 - B_1}{t_2 - t_1} = \frac{0.5 - 0}{2 - 0}$$

$$\frac{dB}{dt} = 0.25 \text{ T/s}$$



Exercise 1:

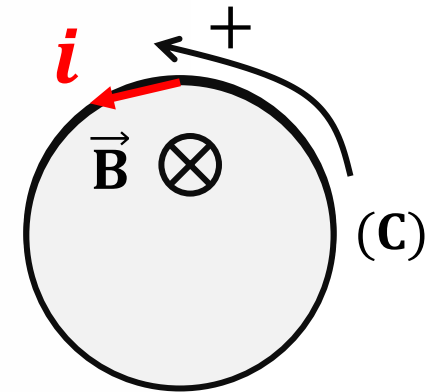
$S = 5 \times 10^{-3} \text{ m}^2$, $r = 10 \Omega$ and of 50 turns.

$$i = \frac{NS}{R} \times \frac{dB}{dt}$$

$$i = \frac{50 \times 5 \times 10^{-3}}{10} \times 0.25$$

$$i = 6.25 \times 10^{-3} \text{ A}$$

Since $i > 0$ the I flow as the positive direction



Doc. 1

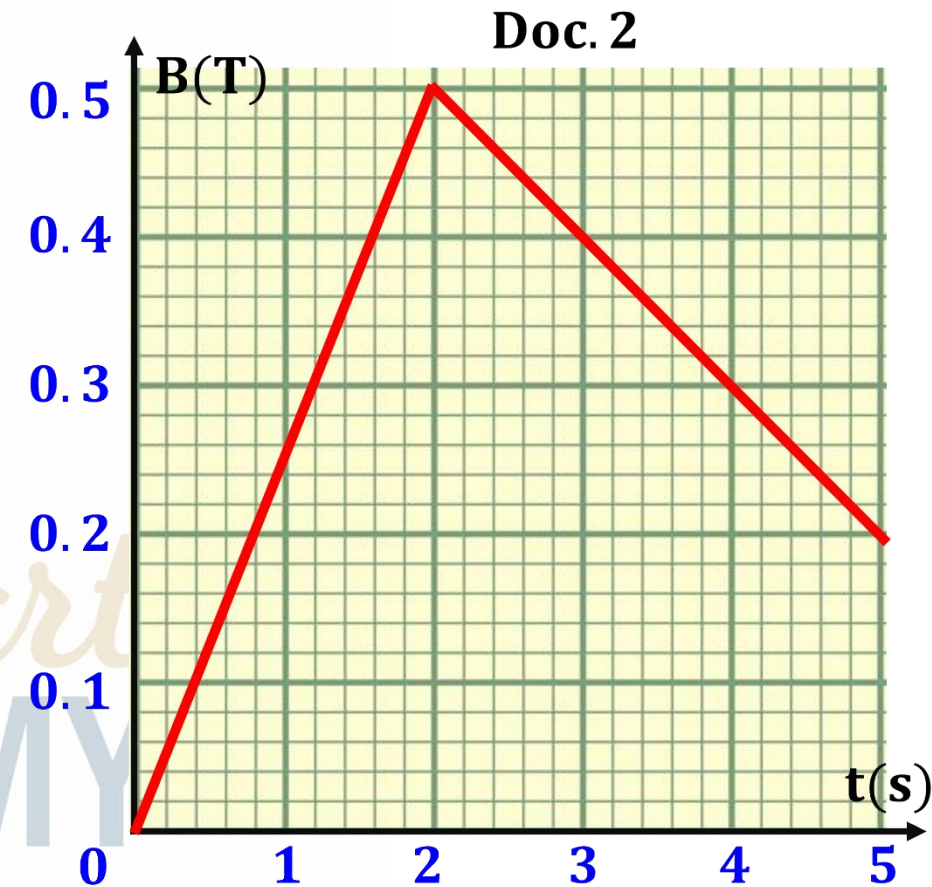
Exercise 1:

$S = 5 \times 10^{-3} \text{ m}^2$, $r = 10 \Omega$ and of 50 turns.

For $2\text{s} \leq t \leq 5\text{s}$

$$\text{slope} = \frac{dB}{dt} = \frac{B_2 - B_1}{t_2 - t_1} = \frac{0.2 - 0.5}{5 - 2}$$

$$\frac{dB}{dt} = -0.1 \text{ T/s}$$



Exercise 1:

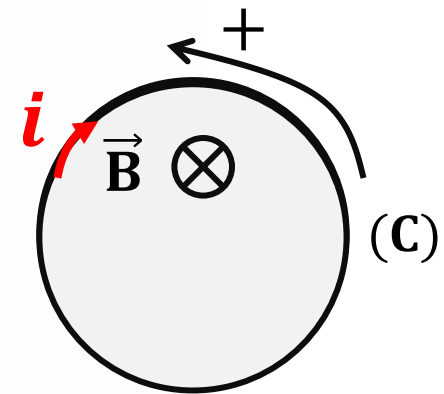
$S = 5 \times 10^{-3} \text{ m}^2$, $r = 10 \Omega$ and of 50 turns.

$$i = \frac{NS}{R} \times \frac{dB}{dt}$$

$$i = \frac{50 \times 5 \times 10^{-3}}{10} \times (-0.1)$$

$$i = -2.5 \times 10^{-3} \text{ A}$$

Since $i < 0$ the I flow in
opposite direction to positive
direction



Doc. 1

Exercise 1:

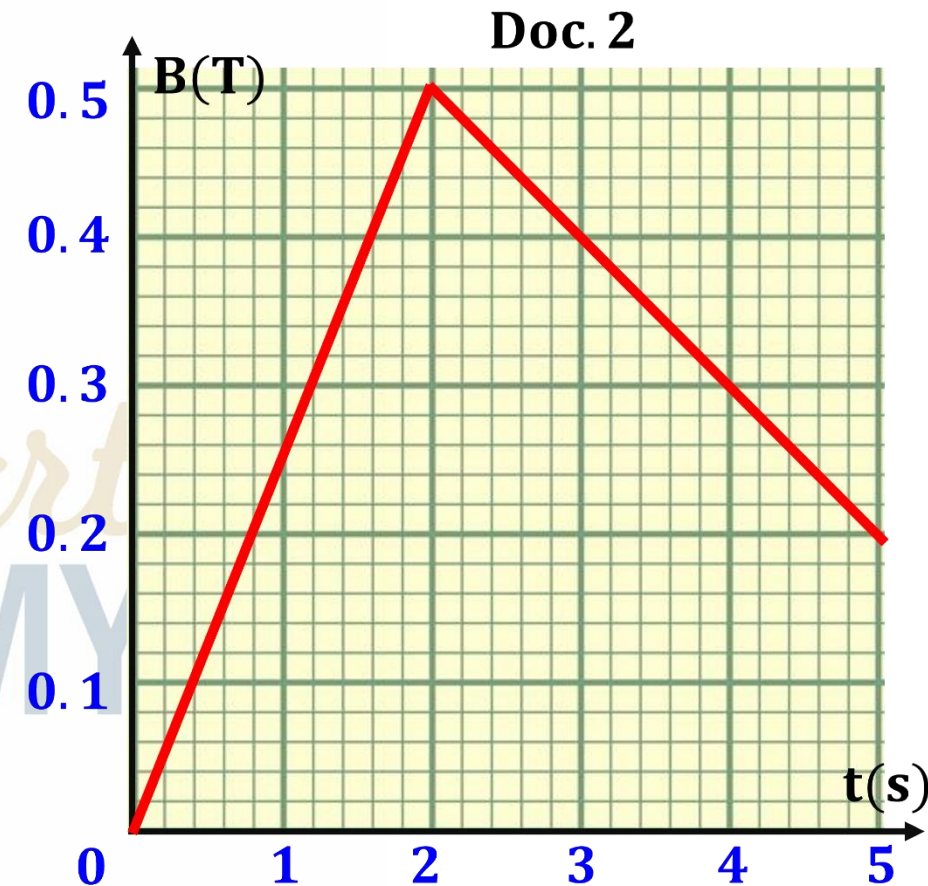
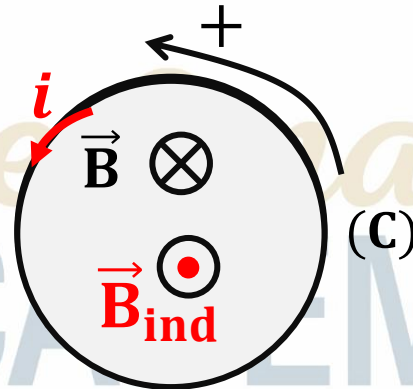
$S = 5 \times 10^{-3} \text{ m}^2$, $r = 10 \Omega$ and of 50 turns.

4. Apply Lenz's law to verify the results obtained in part 3.

For $0 \leq t \leq 2 \text{ s}$

The magnetic field (\vec{B}) increases then \vec{B}_{ind} is in opposite direction.

Using RHR along \vec{B}_{ind} :
 i flow as positive direction



Exercise 1:

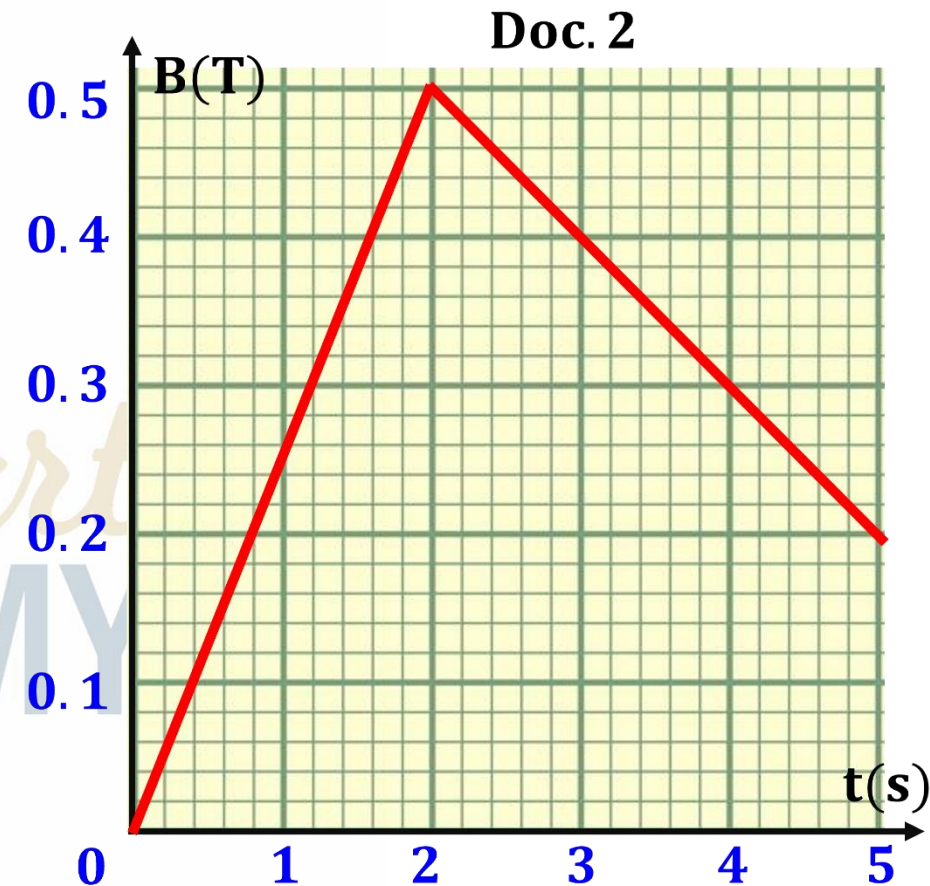
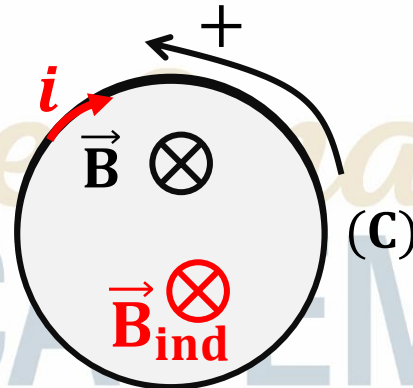
$S = 5 \times 10^{-3} \text{ m}^2$, $r = 10 \Omega$ and of 50 turns.

4. Apply Lenz's law to verify the results obtained in part 3.

For $2\text{s} \leq t \leq 5\text{s}$

The magnetic field (\vec{B}) decreases then \vec{B}_{ind} is in same direction.

Using RHR along \vec{B}_{ind} :
 i flow opposite to positive direction



Exercise 1:

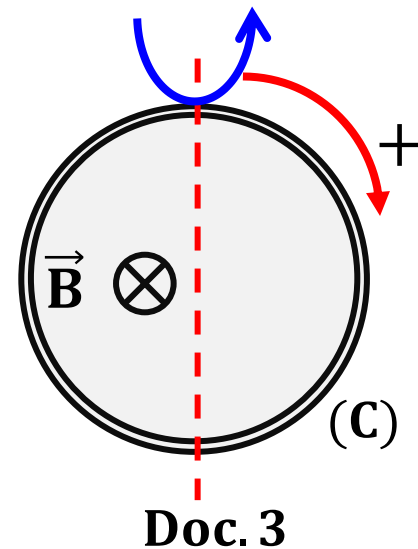
Part 2: Rotation of the coil:

The intensity of the magnetic field \vec{B} is adjusted at $B = 0.5T$. (C) rotates at a constant angular velocity ω about its diameter as shown in document 3.

Let θ be the angle between the normal \vec{n} to the plane of (C) and \vec{B} at an instant t .

1. Knowing that $\theta_0 = 0$ at the instant $t_0 = 0$, show that $\theta = \omega t$.

2. Deduce that the expression of the magnetic flux crossing (C) is given by: $\Phi = NBS \cos(\omega t)$.



Exercise 1:

3. Justify, qualitatively, the existence of an induced e.m.f “e” during the rotation of (C).
4. Determine, in terms of N , S , B , ω and t , the expression of the induced e.m.f “e”.
5. Calculate the value of ω knowing that the maximum induced emf is $0.5V$.

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Exercise 1:

1. Knowing that $\theta_0 = 0$ at the instant $t_0 = 0$, show that $\theta = \omega t$.

(C) rotates at a constant angular velocity ω :

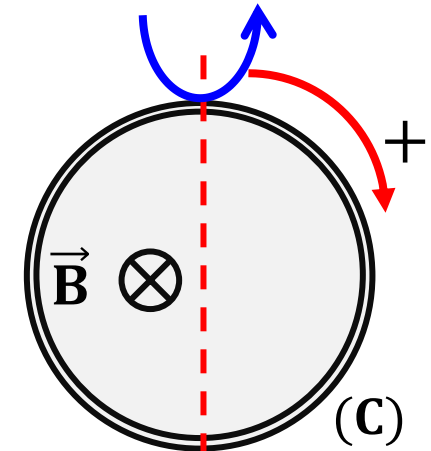
The motion is uniform circular motion:

$$\theta = \omega t + \theta_0$$



$$\theta = \omega t + 0$$

$$\theta = \omega t$$



Doc. 3

2. Deduce that the expression of the magnetic flux crossing (C) is given by: $\phi = NBS \cos(\omega t)$.

$$\phi = NBS \cos(\theta)$$

$$\phi = NBS \cos(\omega t)$$

Exercise 1:

3. Justify, qualitatively, the existence of an induced e.m.f “e” during the rotation of (C).

The magnetic flux (\emptyset) is variable, because of the rotation of the coil around the axis then:

The induced e.m.f “e” exist during the rotation of (C).

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Exercise 1:

4. Determine, in terms of N, S, B, ω and t, the expression of the induced e.m.f “e”.

$$e = -\frac{d\phi}{dt}$$



$$e = -\frac{d(NBS\cos(\omega t))}{dt}$$

$$e = -NBS \frac{d(\cos(\omega t))}{dt}$$

$$e = +NBS\omega \cdot \sin(\omega t)$$

Exercise 1:

5. Calculate the value of ω knowing that the maximum induced emf is $0.5V$.

$$e = +NBS\omega \cdot \sin(\omega t)$$

e is maximum for $\sin(\omega t) = 1$

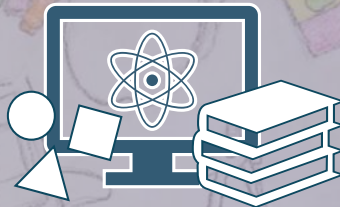
$$e = +NBS\omega$$

$$\omega = \frac{e}{NBS}$$


$$\omega = \frac{0.5}{50 \times 0.5 \times 5 \times 10^{-3}}$$

$$\omega = 4 \text{ rad/s}$$

The End



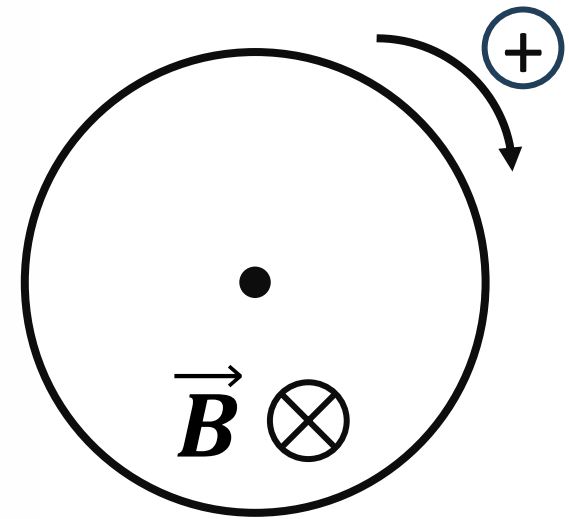


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Exercise 2:

The purpose of this exercise is to determine the direction of the induced current in a circular loop by two different methods. Consider a circular conducting loop of radius $r = 10$ cm and resistance $R = 2 \, \Omega$. The loop is placed in a uniform magnetic field \vec{B} .

The value B of the magnetic field \vec{B} decreases with time according to the relation: $B = -0.04t + 0.8$ (SI).



Exercise 2:

- 1) A current is induced in the loop during the time interval. Justify.
- 2) Apply Lenz's law to specify the direction of the induced current.
- 3) Determine the expression of the magnetic flux crossing the loop as a function of time.
- 4) Deduce the value of the induced electromotive force « e ».
- 5) Deduce the value and the direction of « i ».
- 6) Compare the direction of the induced current obtained in part (2) to that obtained in part (5).

Exercise 2:

**1) A current is induced in the loop during the time interval.
Justify.**

The magnitude of the magnetic field (\vec{B}) changes, then:

The loop is crossed by a variable magnetic flux; therefore:

The loop becomes the seat of induced emf.

The loop forms a closed circuit, then it carries electric current.

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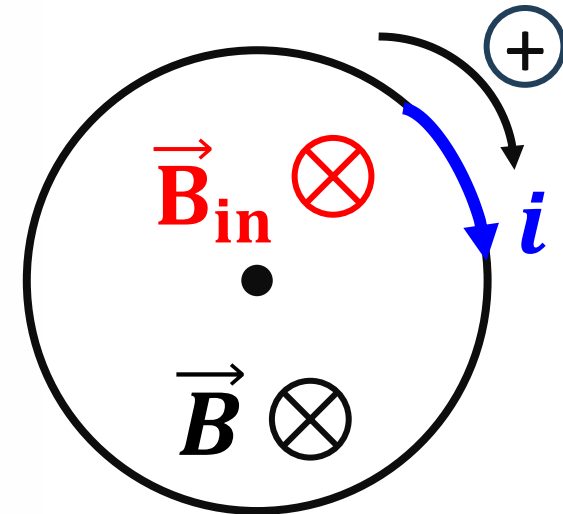
Exercise 3:

2) Apply Lenz's law to specify the direction of the induced current.

The magnitude B of the magnetic field decreases then:

The direction of the induced magnetic field (\vec{B}_{in}) is the same as that of \vec{B} .

According to the RHR, the induced current passes in the loop in the chosen positive sense



Exercise 1:

$$r = 10 \text{ cm}; R = 2 \Omega; B = -0.04t + 0.8$$

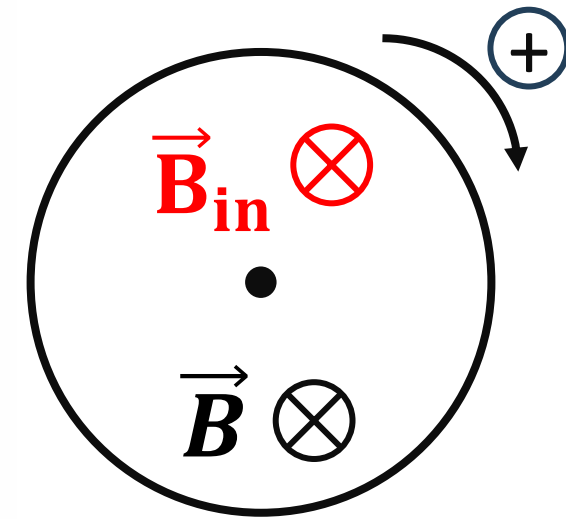
3) Determine the expression of the magnetic flux crossing the loop as a function of time.

$$\phi = B S \cos(\vec{B}, \vec{n}) \Rightarrow \phi = B S \cos(0)$$

$$\phi = B \times \pi r^2 \times (1) \Rightarrow \phi = (-0.04t + 0.8) \times \pi (0.1)^2$$

$$\phi = (-0.04t + 0.8) \times 0.01\pi$$

$$\phi = -4\pi \times 10^{-4}t + 8\pi \times 10^{-4}$$



Exercise 1:

$$r = 10 \text{ cm}; R = 2 \Omega; B = -0.04t + 0.8$$

4) Deduce the value of the induced electromotive force « e ».

$$e = -\frac{d\Phi}{dt}$$

$$e = -\frac{d[-4\pi \times 10^{-4}t + 8\pi \times 10^{-4}]}{dt}$$

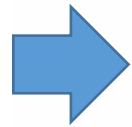
$$e = 4\pi \times 10^{-4} \text{ V}$$

Exercise 1:

$$r = 10 \text{ cm}; R = 2 \text{ } \Omega; B = -0.04t + 0.8$$

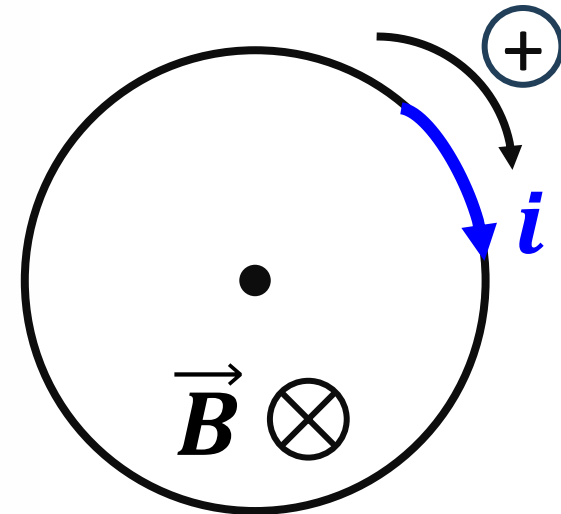
5) Deduce the value and the direction of « i ».

$$i = \frac{e}{R}$$



$$i = \frac{4\pi \times 10^{-4}}{2}$$

$$i = 6.3 \times 10^{-3} \text{ A}$$



$i > 0$, then the current is in the chosen positive sense

Exercise 1:

6) Compare the direction of the induced current obtained in part (2) to that obtained in part (5).

The direction is the same in the two parts.



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The End



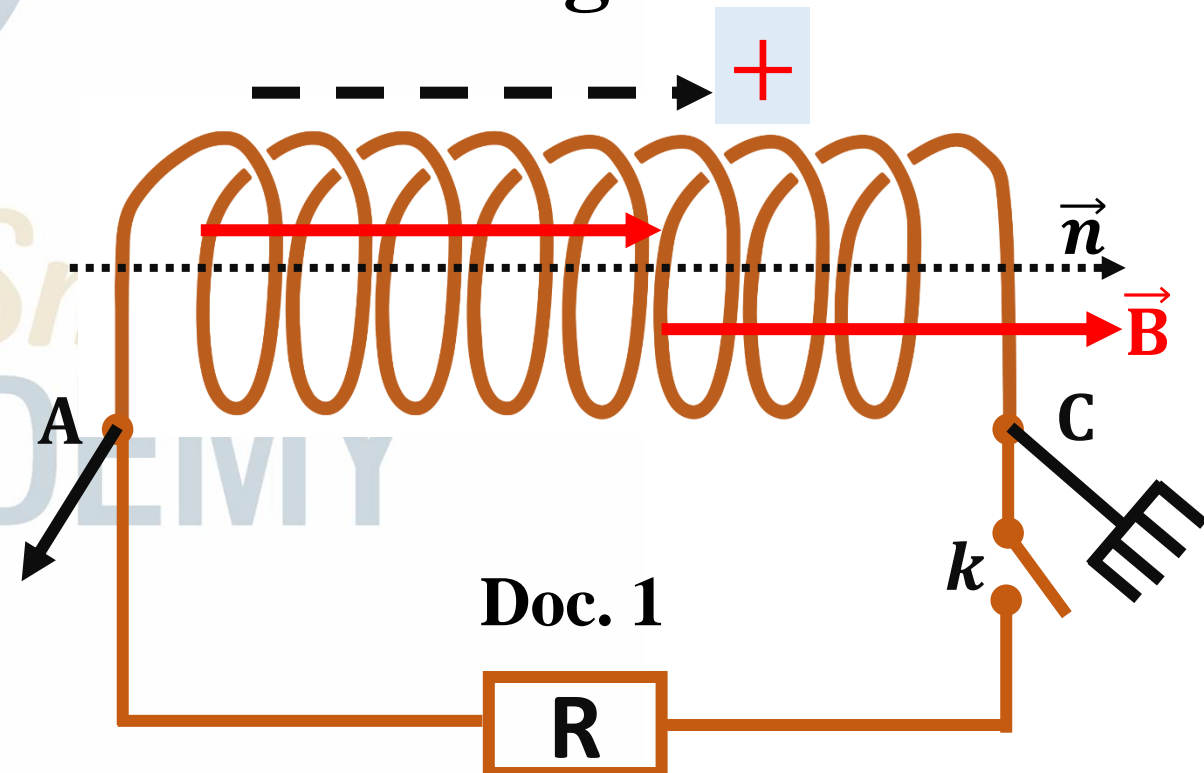


Think then Solve

Exercise 3:

A coil of 1000 turns, and of internal resistance r , is connected to a switch K and a resistor of resistance $R = 8\Omega$. The area of each loop is S . The coil whose axis is horizontal is placed in a uniform magnetic field \vec{B} of magnitude B that varies with time

An oscilloscope is connected across the terminals A and C of the coil to display the voltage u_{AC} .

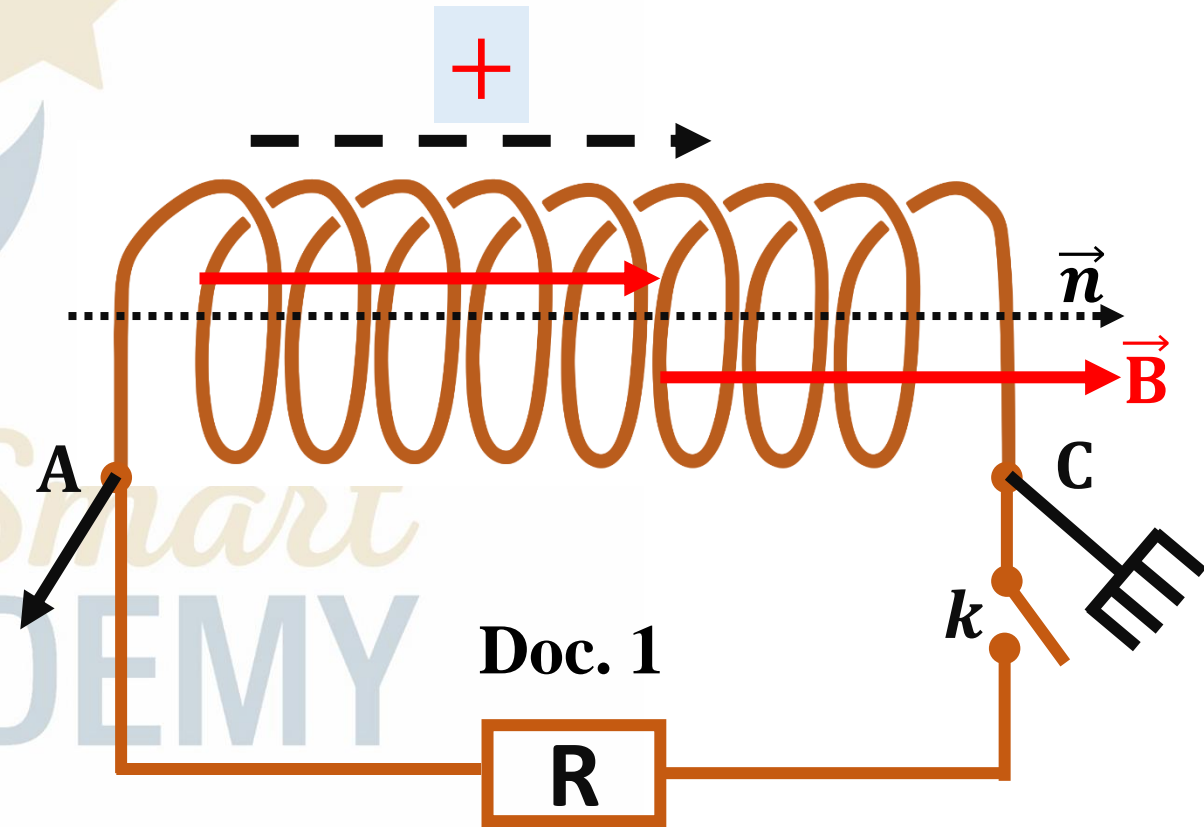


Exercise 3:

In the absence of any voltage, the horizontal luminous line is displayed through the center of the screen of the oscilloscope.

The coil is oriented positively from A to C, and \vec{n} is the normal unit vector to the plane of the loops (Document 1).

The aim of this exercise is to determine the surface area S of each loop and the resistance r of the coil.



Exercise 3:

1) Theoretical Study:

1.1. Determine the expression of the magnetic flux crossing the coil in terms of S and B .

1.2. Determine the expression of the induced electromotive force " e " in the coil in terms of S and $\frac{dB}{dt}$.

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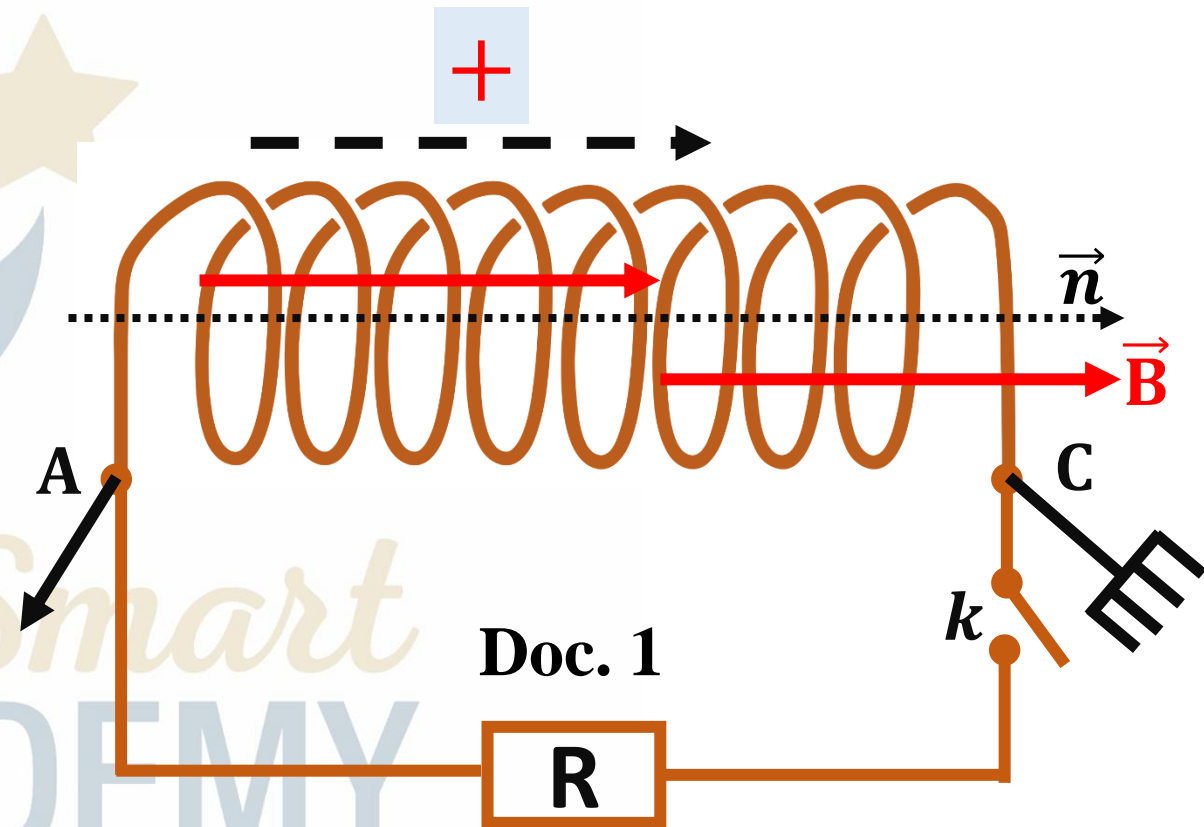
Exercise 3:

1.1. Determine the expression of the magnetic flux crossing the coil in terms of S and B .

$$\Phi = NBS \cos(\vec{n}, \vec{B})$$

$$\Phi = 1000 \times B \times S \times \cos(0)$$

$$\Phi = 1000BS$$



Exercise 3:

1.2. Determine the expression of the induced electromotive force "e" in the coil in terms of S and $\frac{dB}{dt}$.

$$e = -\frac{d\Phi}{dt}$$

$$e = -\frac{d(1000BS)}{dt}$$

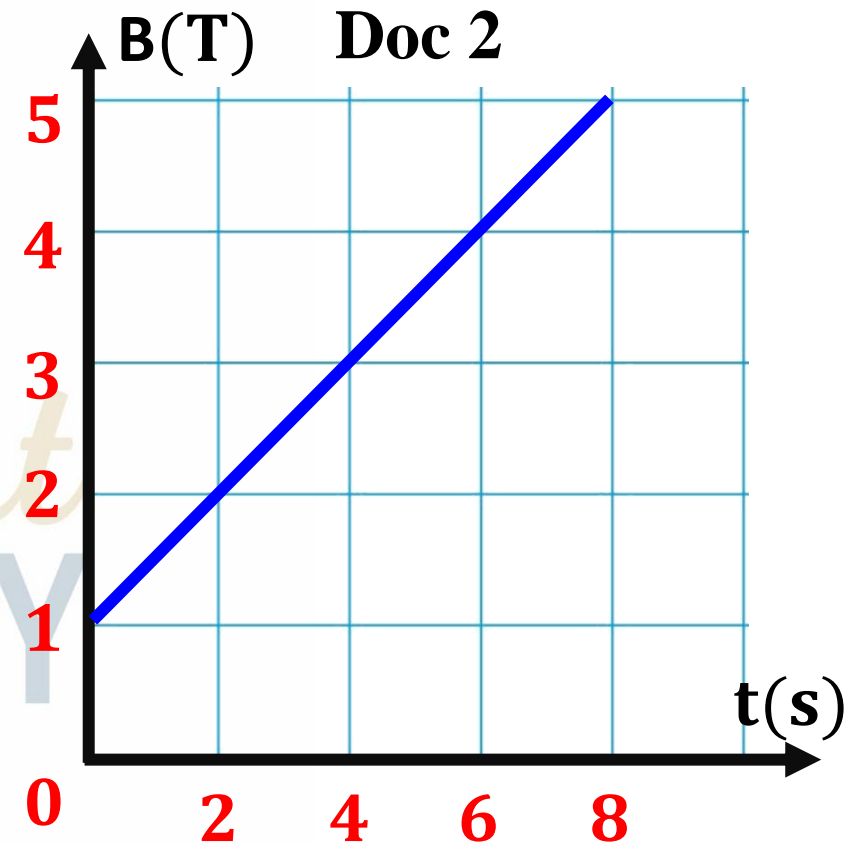
$$e = -1000S \frac{dB}{dt}$$

Exercise 3:

The switch K is opened:

Document 2 represents the magnitude B of the magnetic field \vec{B} as function of time.

2.1. Use document 2, show that the expression of the induced electromotive force "e" in terms of S is $e = -500 \times S$.



Exercise 3:

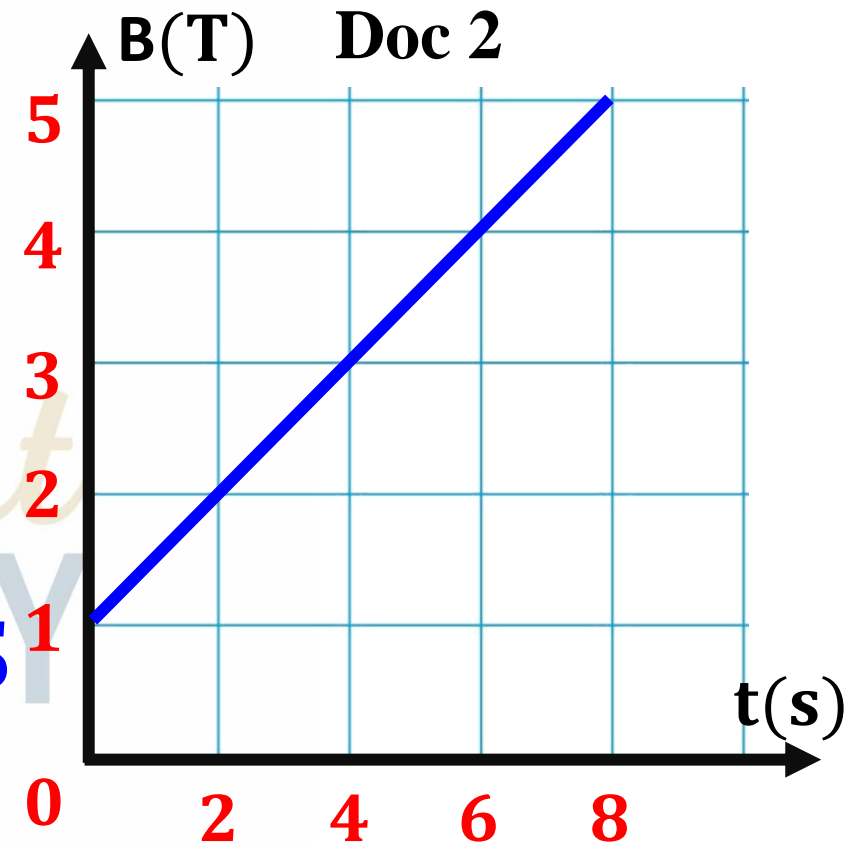
2.1. Use document 1, show that the expression of the induced electromotive force "e" in terms of S is $e = -500 \times S$

The given St. line of equation $B = a \cdot t$, where a is the slope

$$a = \frac{\Delta B}{\Delta t} = \frac{4 - 2}{6 - 2} \Rightarrow a = 0.5 T/s$$

$$e = -1000S \frac{dB}{dt} \Rightarrow e = -1000S \times 0.5$$

$$e = -500S$$



Exercise 3:

2.2. Use document 3, calculate the value of the voltage u_{AC} .

$$u_{AC} = S_V \times y = 0.5 \times 2 \Rightarrow u_{AC} = 1V$$

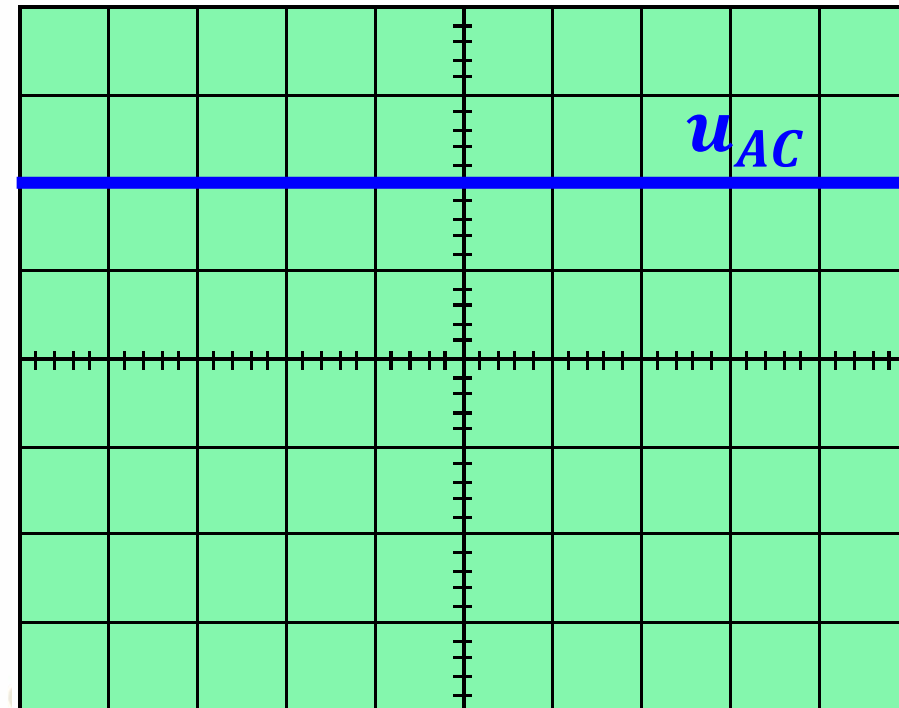
2.3. Deduce that the area of each loop is $S = 20cm^2$.

$$u_{AC} = ri - e$$

Opened circuit then $i = 0$

$$u_{AC} = -e$$

$$1 = -(-500S)$$



$$S_V = 0.5V/div$$

$$S = \frac{1}{500} = 0.002m^2$$

$$S = 20cm^2$$

Exercise 3:

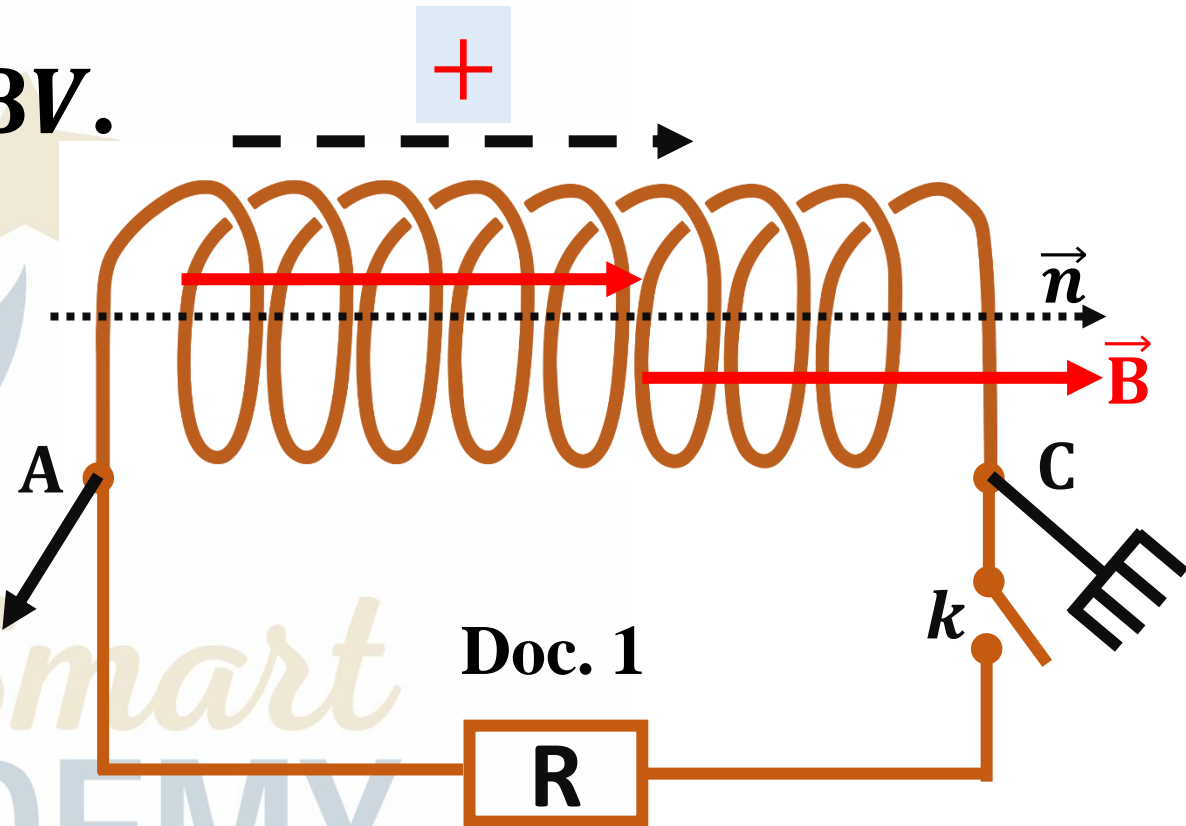
The switch K is closed:

In this case the voltage $u_{AC} = 0.8V$.

3.1. Apply Lenz's law to determine the direction of the induced current.

3.2. Calculate the induced current i .

3.3. Knowing that the induced current is $i = \frac{e}{R+r}$, deduce the value of r .



Exercise 3:

3.1. Apply Lenz's law to determine the direction of the induced current.

Lenz's law states that:

When an e.m.f is induced due to a variation in the magnetic flux, the direction of the induced current is such that its electromagnetic effects oppose the cause that is producing it

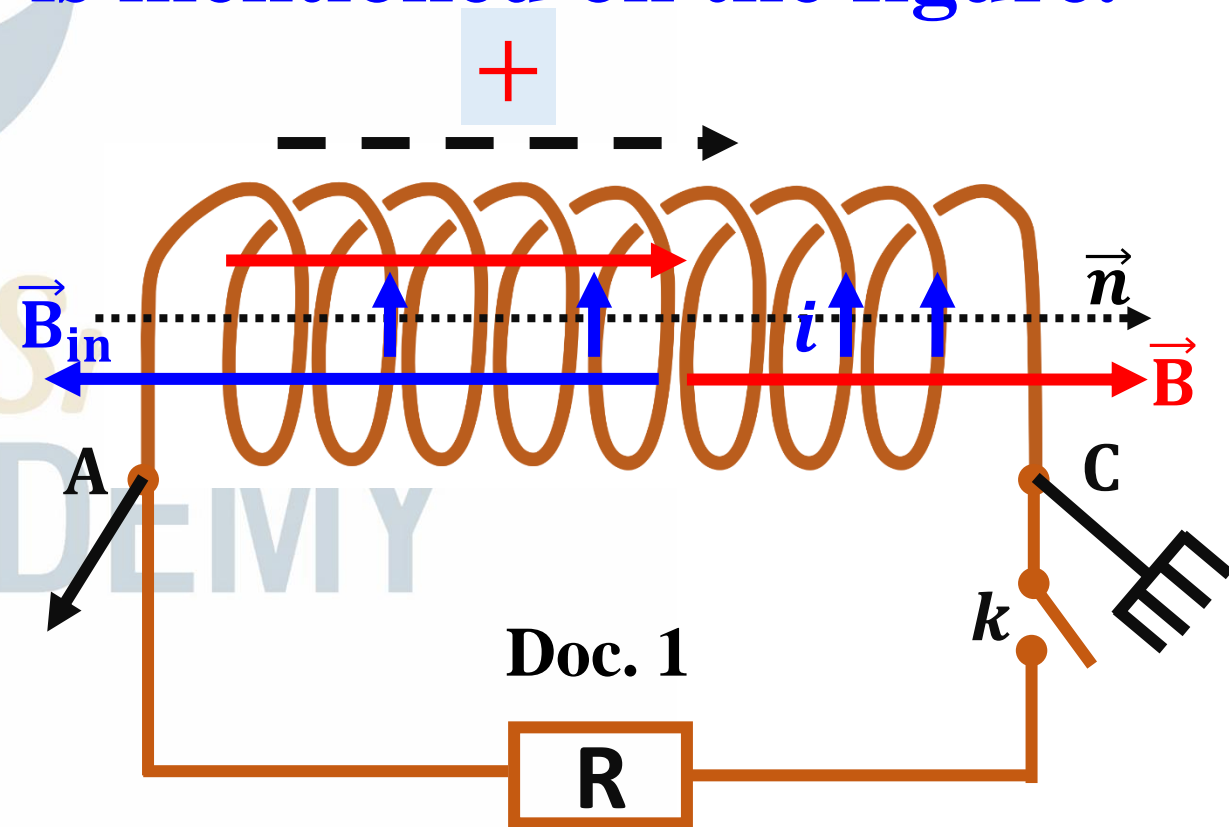
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Exercise 3:

The magnetic field \vec{B} increases then \vec{B}_{in} is opposite to \vec{B} .

Then \vec{B}_{in} directed horizontally due left.

Using RHR the induced current is mentioned on the figure.



Exercise 1:

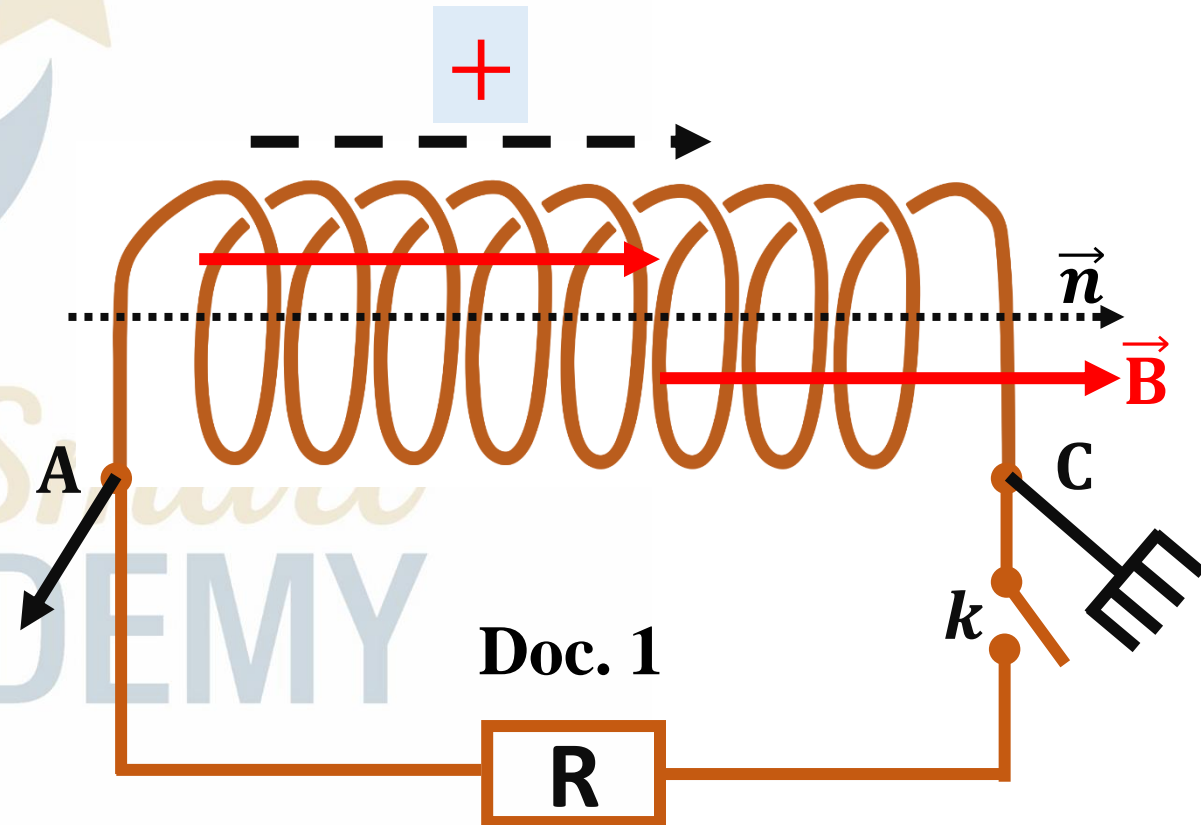
3.2. Calculate the induced current i

Apply Ohm's law of resistor:

$$u_{AC} = Ri$$

$$0.8V = 8i$$

$$i = \frac{0.8}{8} = 0.1A$$



Exercise 1:

3.3. Knowing that the induced current is $i = \frac{e}{R+r}$, deduce the value of r .

$$e = -500S$$

$$e = -500 \times (20 \times 10^{-4})$$

$$e = 1V$$

$$i = \frac{e}{R+r}$$

$$0.1 = \frac{1}{8+r}$$

$$8+r = \frac{1}{0.1}$$

$$8+r = 10$$

$$r = 2\Omega$$

